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ABSTRACT

Through individual interviews with remedial and precalculus university students, five characteristics of the student were identified that influence whether a diagram is used in solving a problem and whether its use is successful. The interviews also suggested a set of subskills that contribute to the overall ability to draw effective diagrams. These were then used in an experimental study to verify their impact on diagram drawing effectiveness. The list of subskills was translated into a series of external control suggestions for guiding the subject's work during an individual interview. Subjects who received these suggestions drew higher quality diagrams than did subjects in the control group. The enhanced quality was particularly apparent in the completeness of the diagrams. (Author)

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# COMPONENTS OF EFFECTIVE USE OF DIAGRAMS IN MATH PROBLEM SOLVING

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Through individual interviews with remedial and precalculus university students, five characteristics of the student were identified that influence whether a diagram is used in solving a problem and whether its use is successful. The interviews also suggested a set of subskills that contribute to the overall ability to draw effective diagrams. These were then used in an experimental study to verify their impact on diagram drawing effectiveness. The list of subskills was translated into a series of external control suggestions for guiding the subject's work during an individual interview. Subjects who received these suggestions drew higher quality diagrams than did subjects in the control group. The enhanced quality was particularly apparent in the completeness of the diagram.

The drawing of diagrams has traditionally been used by individuals as an aid in solving mathematics and science problems. Although mathematics education researchers have not been able to consistently show a link between the use of diagrams and improved problem solving, there is evidence (McKee 1983) of a link between the ability to draw high quality diagrams and the successful solution of problems.

Instructional interventions, which have attempted to improve student's diagram drawing abilities have been largely unimpressive. Such interventions have been of short duration, one to six weeks, and have lacked a theory of what skills, knowledge, beliefs and affective factors contribute to the successful use of diagrams in problem solving (Simon 1986).

## EXPLORATORY STUDY

Remedial mathematics students at the University of Massachusetts, despite frequently being encouraged and at times required to draw diagrams, seem to consistently choose not to draw diagrams in problem situations for which diagrams could be helpful. Observations of this phenomenon led to the

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following questions which motivated this project:

1. What factors affect whether a student chooses to draw a diagram when a diagram could be helpful?

2. What skills and knowledge are required to draw useful diagrams for solving mathematical problems?

An exploratory study was conducted to investigate these two research questions. A clinical interview approach was used in order to investigate, not only the problem solving and diagram work of the subjects, but also the subjects' explanations for their work and their feelings and attitudes about mathematics, problem solving, and the use of diagrams. Examination of the videotapes of these interviews generated two models: a model of the factors which influence the use of diagrams and a model of diagram drawing subskills.

**Model One:** Five characteristics of problem solvers seem to affect whether they choose to draw a diagram and whether they draw useful diagrams. They are:

1. **Understanding of mathematical concepts involved in the problem:** Rote algorithms can often be applied even when conceptual understanding is lacking (e.g., fraction operations). However, an understanding of concepts is often required to figure out an appropriate diagrammatic representation.

2. **Diagram drawing skills and experience:** There are specific skills which are important to successfully create and manipulate a diagram. Some of these skills are also applicable when using other problem solving strategies while others are particular to diagram drawing. (See Model Two, below).

In addition to diagram drawing skills, the experience that the student has with diagram drawing gives the student a sense of how diagram drawing works and what its benefits are. This experience leads to the "metacognitive" skill (Schoenfeld 1983) of thinking to draw a diagram. For example, experienced geometry problem solvers, when faced with problems that ask them to find the size of an angle, proceed to label all of the known angles with numbers or variables, as well as all congruent line segments in hopes of "seeing" the value of the angle in question or a key relationship to it. Without the expectations of how a diagram might help, they would be unlikely to proceed in this way.

3. **Conceptions of mathematics:** The beliefs that students have about mathematics strongly influence the likelihood of their using a diagram. If students believe that mathematics is unrelated to the real world, that it is a

"black box" which can never be understood, and that there is one correct way to do a math problem, then they are unlikely to try to represent the problem diagrammatically in order to explore solution possibilities.

4. **Self-concept in mathematics:** Strongly tied to the students' beliefs about mathematics are their beliefs about themselves as mathematics students. To the extent that the student feels a sense of personal power, a sense of control in mathematics, she is more likely to try to represent the problem in a diagram. If the student feels that the power lies outside of herself, "the teacher hasn't shown me how to do that", or helpless in solving problems, "I'm just not good at word problems", she is unlikely to see any value in drawing a diagram. In fact, the challenge to draw a diagram becomes another frustrating problem to be avoided.

5. **Motivation to solve the problem correctly:** Seemingly obvious, but important to mention, is the student's motivation to solve the problem correctly. For a math anxious student, the motivation may be to finish with the problem (ending the anxiety) rather than to figure out how to solve it correctly. For such a student, diagram drawing appears to be an unnecessary, time-consuming step which only increases the struggle with the problem. Finding a neat algorithm, even if it yields incorrect solutions, more directly serves the goal of finishing with the problem.

**Model Two:** The following are skills and procedures which seem to contribute to successful diagram drawing.

1. Represent all relevant information.
   a. Determine relevant aspects of the problem situation (relevant concepts as well as given information).
   b. Represent everything spatially if possible. (Avoid the use of arithmetic symbols in the diagram (+ - x / =)).
   c. Draw unknown quantities into the diagram keeping track of the unknown (arbitrary) aspects of the diagram.

2. Create one integrated diagram with related parts.
   a. Avoid creating several unrelated diagrams for different aspects of the information.
   b. Operate on the diagram so that the diagram reflects the changed in the problem situation.
   c. As each new piece of information is represented, relate it as much as possible to the already represented information.

3. Label completely.
   a. Label each part of the diagram descriptively (what does it represent).

b. Label all known quantities (include those which become known as you label others).
c. Create and label equal parts wherever possible.

4. Draw multiple representations.
   a. Create alternative representations when unsure how to represent the information. (If unable to decide on an appropriate representation, draw something).
   b. The introduction of new information may necessitate a new diagram.
   c. As the diagram evolves a new diagram may be needed:
      (1) to make a more effective/helpful diagram.
      (2) to give the diagram more accurate proportions.

5. Verbalize about what is represented in the diagram and what needs to be represented.

6. Check the accuracy of the diagram.

## EXPERIMENTAL STUDY

The experimental study was designed to verify that the subskills identified in the exploratory study (Model Two) are indeed important in the creation of useful diagrams. For this purpose a set of external control suggestions was created to encourage subjects to make use of these subskills.

The structure of the Heller and Reif (1984) paradigm, which had been introduced in the area of physics mechanics problems, provided the basis for the methodology employed. The study began with a three-problem pretest of each subject in an individual interview format. The subjects were randomly assigned to either the experimental group or the control group. All subjects returned one week later for treatment sessions which were conducted using a format parallel to that used in the pretest. In the treatment sessions, the control group repeated the pretest procedure, while the experimental group received orally from the experimenter the suggestions that encouraged the subskills from Model Two. Although the experimenter's interventions were limited to those listed, the timing of his suggestions were based on his control knowledge, knowledge of when particular suggestions were appropriate.

During the pretest and treatment sessions, subjects were asked to draw the most complete and useful diagrams that they could. Subjects were not asked to get an answer to the problem.

Fourteen volunteers from precalculus classes at the University of Massachusetts were paid to participate in this study. As in the exploratory study, standard algebra problems which can be solved with a diagram approach were used. The six pretest and treatment problems were selected from McKee's study (1983).

The scoring criteria were based on McKee's four measures: type, completeness, labeling, and accuracy. Difference scores (treatment score - pretest score) were computed reflecting improvement from the first session to the second. The differences for each group were compared for each measure and for the total score using Student t-tests.

The results, shown in the table below, indicate a significantly greater improvement for the experimental group than for the control group based on total scores ($p < .01$). This suggests that at least some of the subskills contained in the model are lacking in the experimental population and contribute to the creation of useful diagrams. Looking at individual measures, we find that the only significant difference exists in the category of diagram completeness ($p < .01$).

TABLE 1: Experimental Versus Control Groups

| Measure | Exp. Group | Cont. Group | p-value | signif. |
|---|---|---|---|---|
| Total | 3.40 (1.35) | 1.45 (1.04) | .005 | ** |
| Type | .38 ( .50) | .02 ( .06) | .054 | |
| Complete | 1.40 ( .74) | .55 ( .30) | .008 | ** |
| Labeling | 1.19 ( .94) | .74 ( .71) | .166 | |
| Accuracy | .45 ( .33) | .14 ( .35) | .058 | |

* = significant, $p < .05$
** = highly significant, $p < .01$

The measures for type of diagram (M1) and accuracy of diagram, while not significant were close to significant ($p = .054$ and $p = .058$ respectively), indicating that a difference may reveal itself in a study with a larger number of subjects. Type, the ability to draw a schematic representation of the mathematical quantities involved rather than an illustration of the problem setting, did not prove to be a significant difficulty for most of the subjects. Possible differences in improvement for type (M1) were likely depressed due to the ceiling effect of scores on both the pretest and treatment problems (Exp. Group -- pretest = 2.64, treatment = 3.00 and Control Group -- pretest = 2.76, treatment = 2.79 out of a possible 3 points).

Experimental subjects showed no significant difference over control subjects on labeling. This may suggest that the process of labeling effectively may be a more complex process which is not generally improved by making external suggestions.

Both the exploratoy study and the experimental study have contributed to the conclusion that diagram drawing is a complex ability which is dependent upon the student's understanding of mathematical concepts, his self-concept in mathematics, and his beliefs about mathematics, as well as a host of general and specific skills. While high levels of functioning on these factors are important for diagram drawing success, it is also possible, and worth investigating, that effective instruction in diagram drawing, conversely, can contribute to improvement on these factors. Thus, diagram drawing may be a useful vehicle through which students' experiences with mathematics can change in a positive direction.

## REFERENCES


Heller, J. and Reif, F. (1984) Prescribing effective human problem-solving processes: problem description in physics, Cognition and Instruction, 2, 177-216.

McKee, L.D. (1983) Figure-drawing ability in solving mathematical problems. (Doctoral dissertation, University of Georgia).

Schoenfeld, A.H. (1985) Metacognitive and epistemological issues in mathematical understanding. In E.A. Silver (Ed.), Teaching and Learning Mathematical Problem Solving: Multiple Research Perspectives (pp. 361-379). Hillsdale, N.J.: Lawrence Erlbaum Associates.

Simon, M.A. (1986) An external control study of diagram drawing skills for the solution of algebra word problems by novice problem solvers. (Doctoral dissertation, University of Massachusetts).